Polinomio di Taylor

Data $f(x)$ derivabile m volte in $x_0 \in \mathrm{Dom}\, f$.

Il polinomio di Taylor di ordine m centrato in x_0 è:

$$F(x_0) + f'(x_0)(x-x_0) + \frac{F''(x_0)}{2!}(x-x_0)^2 + \frac{F''(x_0)}{3!}(x-x_0)^3 + \dots + \frac{F^{m}(x_0)}{m!}(x-x_0)^m.$$

TAVOLA DEGLI SVILUPPI DI TAYLOR DELLE FUNZIONI ELEMENTARI PER $x \to 0$.

ARGOMENTO

$$\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!} + o(x^n) \\
\sin x &= x - \frac{x^3}{6} + \frac{x^5}{5!} + \dots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + o(x^{2n+2}) \\
\cos x &= 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \dots + \frac{(-1)^n}{(2n)!}x^{2n} + o(x^{2n+1}) \\
\tan x &= x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + o(x^{10}) \\
\sinh x &= x + \frac{x^3}{6} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \\
\cosh x &= 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\
\tanh x &= x - \frac{x^3}{3} + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2835}x^9 + o(x^{10}) \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + \dots + x^n + o(x^n) \\
\log(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{(-1)^{n+1}}{n}x^n + o(x^n) \\
\arctan x &= x - \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{(-1)^n}{2n+1}x^{2n+1} + o(x^{2n+2}) \\
\operatorname{arctanh} x &= x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2n+1}}{2n+1} + o(x^{2n+2}) \\
(1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{6}x^3 + \dots + \binom{\alpha}{n}x^n + o(x^n)
\end{aligned}$$

$$\left[-\ln(\cos x)\right]'$$

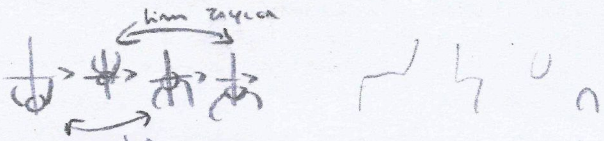
con

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)\cdot\dots\cdot(\alpha-n+1)}{n!} \qquad \frac{\alpha\cdot(\alpha-1)\cdot\dots(\alpha-(m-1))}{m!}$$

CASISTICHE PARTICOLARI:

- $\lim (z)^{(b)} = \lim e^{\log z^b} = \lim e^{b \log z} = e^{\boxed{\lim b \log z}}$ — tramite Taylor.

- Data $F(x)$ in intorno è:



- **NB** Si possono usare gli sviluppi di Taylor anche con $\lim_{n\to\infty} \sin\frac{1}{n}$ purché l'argomento tenda a **zero**